ground water

Analytical and Numerical Models to Explain Steady Rates of Spring Flow

by Susan K. Swanson[1] and Jean M. Bahr[2]

Abstract

Flow from some springs in former glacial lakebeds of the Upper Midwest is extremely steady throughout the year and does not increase significantly after precipitation events or seasonal recharge. Analytical and simplified numerical models of spring systems were used to determine whether preferential ground water flow through high-permeability features in shallow sandstone aquifers could produce typical values of spring discharge and the unusually steady rates of spring flow. The analytical model is based on a one-dimensional solution for periodic ground water flow. Solutions to this model suggest that it is unlikely that a periodic forcing due to seasonal variations in areal recharge would propagate to springs in a setting where high-permeability features exist. The analytical model shows that the effective length of the aquifer, or the length of flowpaths to a spring, and the total transmissivity of the aquifer have the greatest potential to impact the nature of spring flow in this setting. The numerical models show that high-permeability features can influence the magnitude of spring flow and the results demonstrate that the lengths of ground water flowpaths increase when high-permeability features are explicitly modeled, thus decreasing the likelihood for temporal variations in spring flow.

Introduction

Springs found in former lakebed wetlands in the glaciated Upper Midwest often provide a consistent source of mineralized water needed to sustain threatened wetland plant communities. For example, they are commonly associated with calcareous fens, one of the rarest wetland plant communities in North America (Eggers and Reed 1997). Flow from some springs in this region is extremely steady and does not increase significantly after precipitation events or seasonal recharge (Figure 1) (Domber 2000; Swanson 2001; Anderson 2002). Such steady spring flow can contribute to the generally stable water levels that are characteristic of fens (Mitsch and Goselink 1993). Therefore, identifying hydrogeologic properties that contribute to steady spring flow and predicting the response of springs to stresses will enable us to better manage threatened communities and restore native biodiversity.

Springs emanating from shallow flow systems, particularly in fractured-carbonate or karst environments, often exhibit highly variable discharge in response to individual recharge events, as well as to seasonal patterns of recharge (Atkinson 1977; Dreiss 1982; Avery et al. 1999). Springs with extremely steady rates of flow are not widely documented in the literature. Historically, Bryan (1919) explained steady spring flow (i.e., spring flow not subject to seasonal or shorter-term temporal fluctuations) by faults that create conduits for the discharge of what he called "deep-seated ground water." Halihan and Wicks (1998) show that a constricted passage in an aquifer containing large voids can generate a steady response in well or spring hydrographs. Recent work, summarized in this paper, suggests that shallow sandstone aquifers can also generate springs with steady flow, even in areas where seasonal or higher frequency recharge occurs. Steady flow in such a system can result from diffuse recharge through unlithified deposits or sandstone, followed by focused flow through thin, laterally extensive, high-permeability zones in sedimentary bedrock. To test this conceptual model, we used (1) an analytical model and (2) numerical models that are

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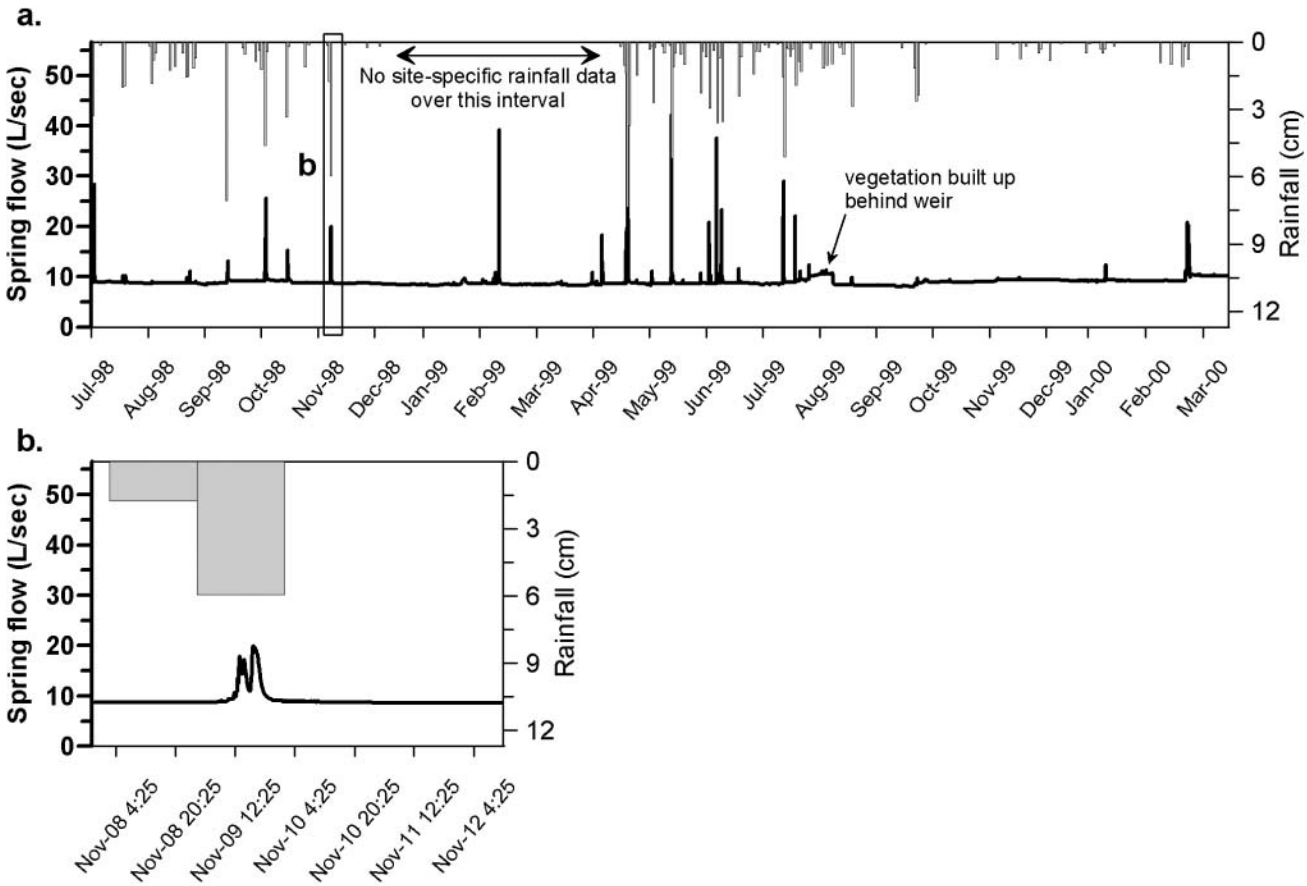

Figure 1. (a) Spring flow record for a typical spring in the Nine Springs Wetlands. Spring flow was calculated using a stage-discharge relationship for a weir designed by Schwar (2002). (b) Portion of the record showing a spike in flow that coincides with a rainfall event. The spike subsides shortly after the rain ends and is not associated with a recession curve that would indicate an event-induced increase in discharge from the spring.

representative of a setting with springs that exhibit extremely steady rates of flow.

Previous analytical approaches to modeling spring flow have concentrated on reproducing the response of large springs (> 300 to 3000 L/s) with highly variable discharge rates in fractured-carbonate and karst environments. For example, Atkinson (1977), Bhar and Mishra (1997), Avery et al. (1999), and many others have used a decreasing exponential or power law to model spring flow recession curves, and Dreiss (1982) used a deconvolution method to predict karstic spring unit hydrographs for storm responses.

In an evaluation of seasonal variations in discharge from an aquifer in volcanic rock, Manga (1996, 1997) successfully used a one-dimensional periodic solution to simulate the magnitude and lag in spring-fed streamflow following annual snowmelt in the Oregon Cascades. We applied a similar model in this study based on the assumptions that seasonal recharge is diffuse rather than focused and can be approximated by a sinusoidal function. The purpose of applying the analytical model was to test whether the solution can explain the absence of detectable event based or seasonal spring flow variations in a hydrogeologic setting where the length scale of the aquifer is comparable to, or shorter than, those in the Cascades, but in which preferential ground water flow through thin, laterally extensive, high-permeability zones may influence spring flow characteristics.

While the analytical solution provides a means to assess the steady nature of spring flow, it cannot explicitly test the effects of high-permeability features on the magnitude of focused spring flow. Finite-difference codes, such as MODFLOW (McDonald and Harbaugh 1988) used in this study, provide a means to incorporate such geologic features into a second class of numerical models designed to assess the impact of high-permeability features on spring flow. The numerical models were designed to incorporate the salient features of the hydrogeologic system under study, but they are not site-specific. The objective was to include enough detail to simulate realistic conditions, but not so much that cause and effect relations are obscured. This approach has been shown to be successful in the identification of general features and properties that help to explain overall ground water flow characteristics (Anderson and Bowser 1986; Morgan and Jones 1999).

Study Area

Spring Characteristics

The Nine Springs Wetlands in Dane County, Wisconsin (Figure 2) contain a number of major springs that discharge water at rates of 3 to 30 L/s. These are boiling springs with sandy bottoms that are found near the margins of the wetlands that surround Nine Springs Creek. Many smaller

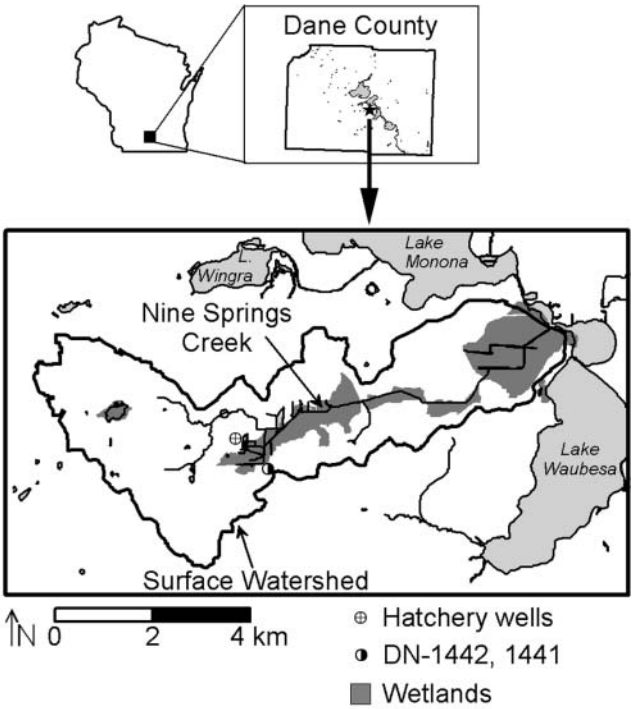

Figure 2. Location of the Nine Springs Wetlands in southern Wisconsin. Wetlands are represented by dark shading and lakes are represented by light shading.

springs and numerous seeps also exist within the wetlands. Discharge from six major springs and/or spring complexes (localized discharge with two or more individual boils and/or flowing wells) was monitored for 3 yr, from 1998 to 2001 (Swanson 2001; Schwar 2002). Figure 1a shows a representative record of flow from one of the springs. Spring flow at this site remained steady at ~9 L/s for 3 yr, with no obvious seasonal or interannual variations. Spikes in flow that coincide with rainfall events are due to localized surface water runoff into the spring pool behind the weir. These subside shortly after the rain ends and are not associated with recession curves that would indicate event-induced increases in discharge from the spring (Figure 1b).

Hydrostratigraphy

Figure 3 summarizes the regional geology and hydrostratigraphy. The bedrock in the area consists of clastic sedimentary rocks that overlie undifferentiated igneous and metamorphic rocks. The sedimentary units dip regionally only 2 to 3 m/km to the south (Cline 1965). However, the bedrock surface is heavily dissected and forms an east-west trending buried bedrock valley below the Nine Springs Wetlands (Olcott 1972a).

Bradbury et al. (1999) defined three regional aquifers and one aquitard in the Dane County area—the surficial unlithified aquifer, the upper bedrock aquifer, the lower bedrock aquifer, and the Eau Claire Aquitard. The lower bedrock aquifer, composed primarily of the Mt. Simon Formation, overlies crystalline rocks that form the base of the aquifer system. Within much of Dane County, the Eau Claire Formation contains a shale and siltstone facies that constitutes a leaky confining unit (the Eau Claire Aquitard) between the lower bedrock aquifer and the upper bedrock aquifer. The upper bedrock aquifer includes all of the bedrock units that are younger than the Eau Claire Formation (Bradbury et al. 1999). The uppermost bedrock unit within the buried bedrock valley in the Nine Springs area is probably the Wonewoc Formation or the Tunnel City Group. The Tunnel City Group, St. Lawrence Formation, and the Jordan Formation are the uppermost units, respectively, with increasing distance from the buried bedrock valley (Olcott 1972b). The unconfined unlithified aquifer, which mantles the bedrock in the uplands and fills the

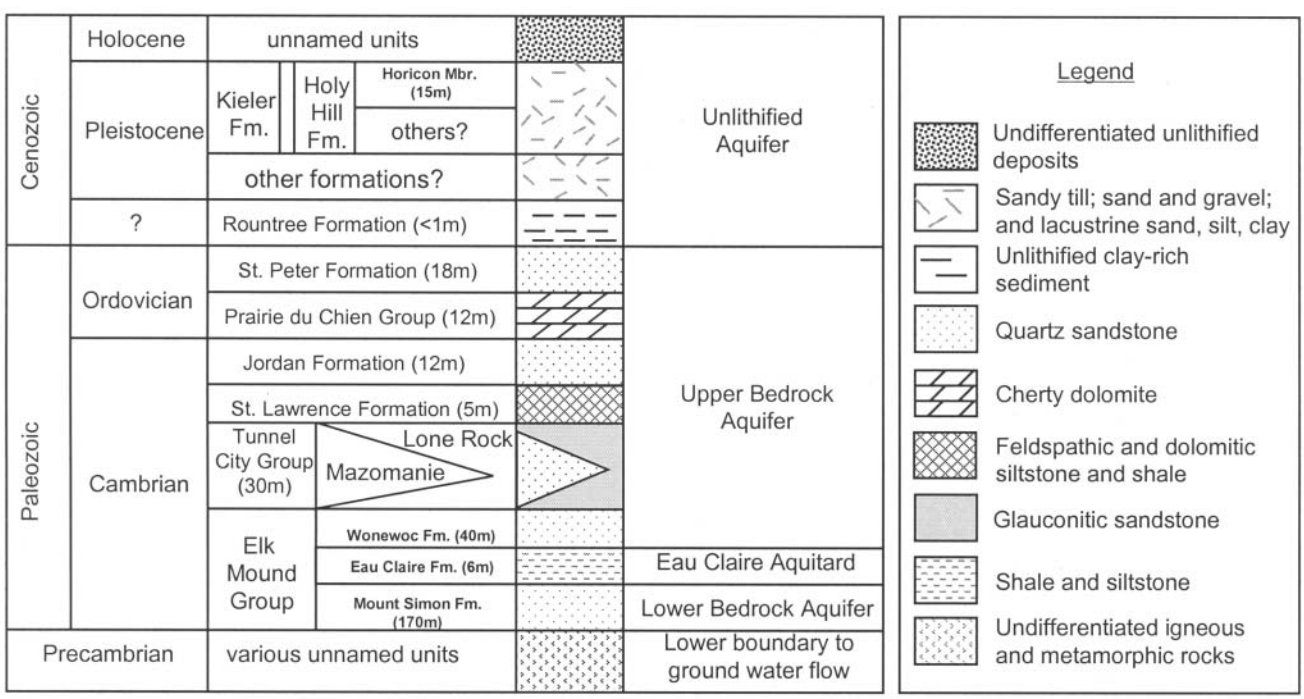

Figure 3. Stratigraphic section for the Nine Springs area (modified from Clayton and Attig 1997). Average unit thicknesses are included in parentheses. Section also shows the regional hydrostratigraphy as defined by Bradbury et al. (1999).

bedrock valley, consists of sandy till, outwash, and glaciolacustrine materials belonging to the Horicon Member of the Holy Hill Formation (Clayton and Attig 1997; Bradbury et al. 1999).

Results of borehole geophysics and slug tests, conducted in cooperation with the Wisconsin Geological and Natural History Survey, provide evidence for high-permeability zones in the Tunnel City Group and at its contacts with the Wonewoc and St. Lawrence formations. Geophysical logs, including natural gamma, fluid temperature, and fluid resistivity, were collected at test well DN–1442 and two Wisconsin Department of Natural Resources (WDNR) wells located at the Nevin State Fish Hatchery (Figure 2). Because the two hatchery wells (WDNR#7 and WDNR#8) are flowing wells (up to 25 L/s), an impeller flowmeter also provided a relative measure of vertical flow, thus identifying where water enters the boreholes. Comparison of the geophysical logs from DN–1442 with those from WDNR#7 and WDNR#8 shows good correlations among peaks in the natural gamma logs (Figure 4). The Tunnel City Group is generally described as a fine- to medium-grained, cross-bedded, dolomitic and, in some places, glauconitic sandstone (Odom 1978). The similarities among the natural gamma logs of the three wells, which are ~1 km apart, suggest that this unit is composed of beds of variable clay content that are laterally continuous in the area. Some of these beds are as thin as 0.3 m or less.

Slug tests in DN–1442, which is cased through the unlithified materials and open to the St. Lawrence Formation, the Tunnel City Group, and the Wonewoc Formation, were conducted at discrete intervals using a straddle-packer assemblage designed to isolate an interval of the aquifer with a maximum length of 1.1 m. Tests were conducted continuously over the entire section of the St. Lawrence Formation and Tunnel City Group (36 m) and within the upper 6 m of the Wonewoc Formation (Swanson 2001). Results show that hydraulic conductivity estimates range over three orders of magnitude, from 2.6E–04 to 4.0E–01 cm/s, for the glauconitic sandstone of the Tunnel City Group; however, the hydraulic conductivity distribution is not unimodal. Zones of relatively high hydraulic conductivity exist directly above or below zones of low hydraulic conductivity without obvious transition zones at the scale of the test intervals. Two-thirds of the intervals tested yielded hydraulic conductivities < 1.0E–02 cm/s. For purposes of hydrostratigraphic characterization, the remaining intervals with hydraulic conductivities > 1.0E–02 cm/s are considered high-permeability zones within the aquifer. The geometric mean hydraulic conductivity (K_{gm}) of the high-permeability zones is 7.6E–02 cm/s, while the K_{gm} of the remaining sandstone is 9.3E–04 cm/s.

Comparing the hydraulic conductivity test results to the fluid temperature and fluid resistivity logs for DN–1442 and the fluid temperature, fluid resistivity, and flowmeter logs for WDNR#7 and WDNR#8 shows that many of the high-permeability zones in the Tunnel City Group correspond to locations of sharp changes in fluid temperature, resistivity, and/or flow rate (Figure 4). Sharp changes in these logs can be qualitative indicators of features that allow preferential ground water flow.

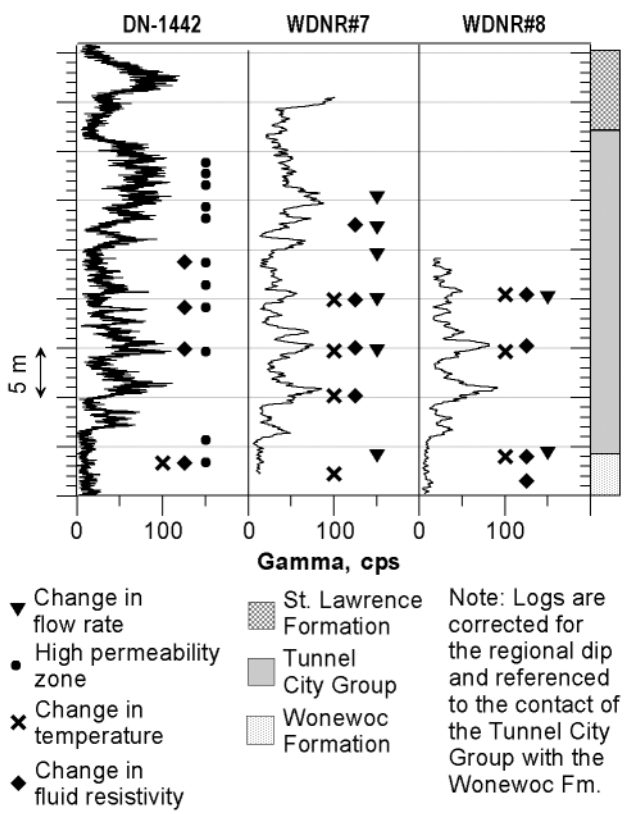

Figure 4. Natural gamma logs and summary of correlatable hydrogeologic properties, changes in fluid properties, and changes in flow rate among DN–1442 and hatchery wells WDNR#7 and WDNR#8.

Ground Water Recharge

Total annual recharge estimates for Dane County range from 15.2 to 27.9 cm/yr (Swanson 1996). Ground water recharge is thought to occur primarily during two periods, spring and late fall, in response to precipitation, snowmelt, evapotranspiration, runoff, and changes in soil moisture (Swanson 1996; Dripps et al. 2000).

Conceptual Model

The conceptual model, developed on the basis of the field observations discussed previously, is that ground water preferentially flows along high-permeability zones due to limited cementation along bedding plane partings or erosional disconformities, horizontal fractures, or coarse beds in the upper bedrock aquifer. Springs form where these zones are truncated by a buried bedrock valley and where the potentiometric surface exceeds the land surface (Figure 5). We assume that the sedimentary aquifers and the high-permeability zones are laterally continuous.

Two alternative conceptual models were initially considered as plausible hydrogeologic systems that might contribute to the steady rates of spring flow observed in this hydrogeologic setting. The first conceptual model suggests that ground water flows primarily through the unlithified aquifer and springs develop where glaciolacustrine deposits coarsen or thin at the margins of buried bedrock valleys and where the water table intersects the land surface. However, it seems likely that ground water discharging primarily from the unlithified aquifer would exhibit variations in flow in response to seasonal variations in ground water recharge.

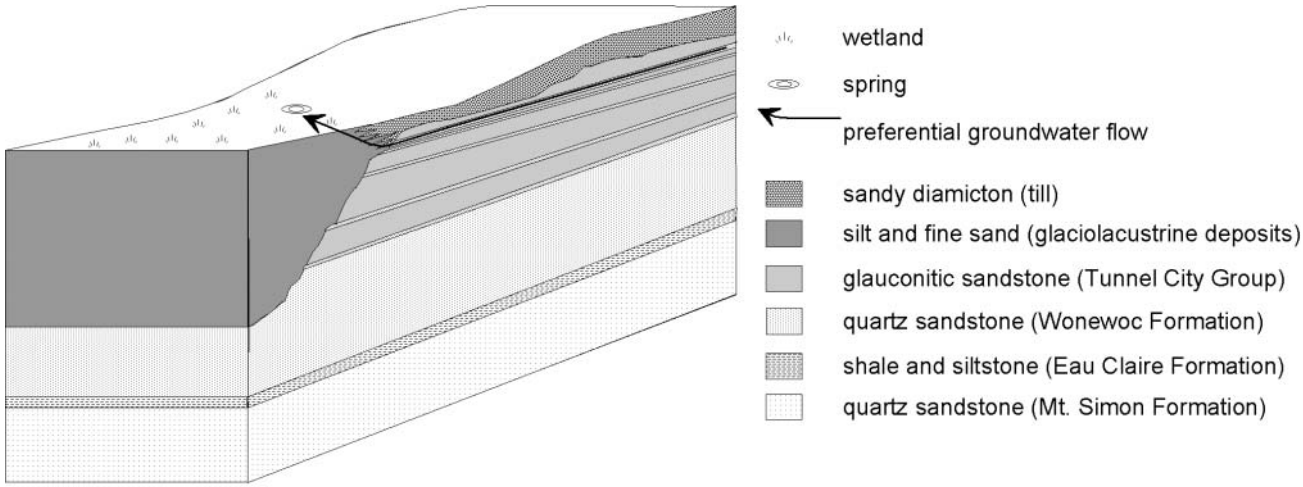

Figure 5. Conceptual model of springs. Springs form where high-permeability zones in a shallow sandstone aquifer are truncated by a bedrock valley and where the potentiometric surface exceeds the land surface.

In addition, hydrogeochemical results suggest that ground water discharging to several representative springs in the Nine Springs Wetlands travels primarily through the upper bedrock aquifer rather than through the unlithified aquifer or the lower bedrock aquifer (Swanson et al. 2001). A second conceptual model is similar to the model proposed by Bryan (1919)—spring flow is supplemented with water discharging from a semiconfined lower bedrock aquifer, thus contributing to steady spring flow. Such conditions could develop where the Eau Claire Aquitard is locally absent or leaky, thereby allowing upward flow of water from the lower bedrock aquifer (Hunt et al. 2001). However, 4.5 to 6 m of shale were encountered while drilling test well DN–1441 located near the margin of the buried bedrock valley and within 300 m of several large spring complexes (Figure 2). Water level monitoring in this well, which is open only to the upper 6 m of the lower bedrock aquifer, and in DN–1442, open to the upper bedrock aquifer, shows that a head drop of ~18 m occurs across the shale. Thus, a positive hydraulic gradient (indicating downward flow) exists between the upper bedrock aquifer and the lower bedrock aquifer. The lower heads in the lower bedrock aquifer are the result of municipal pumping in central Dane County (Bradbury et al. 1999). The large difference in head implies that the Eau Claire Aquitard effectively restricts flow between the upper bedrock aquifer and the lower bedrock aquifer in this region.

Analytical Models of Periodic Ground Water Flow

Jacob (1950) was the first to extend periodic solutions from heat flow to ground water flow for use in quantifying aquifer response to tidal fluctuations. Many others have since applied periodic solutions to problems of tidal fluctuations, ground water recharge, evapotranspiration, spring-dominated streamflow, and pumping (Carr and Van der Kamp 1969; Van der Kamp 1973; Davis and Peck 1986; Bundschuh 1993; Townley 1995; Manga 1996, 1997). Bundschuh (1993) investigated the influence of annual periodic surface temperatures on temperatures of spring water; however, in doing so, Bundschuh specified spring fluxes and considered only simplified aquifer systems. Manga (1996, 1997) related periodic solutions to flow in spring-dominated streams and showed that a periodic solution adequately explains the amplitude of the fluctuations in streamflow, as well as the lag in the maximum flux observed in four streams. Although the simplified model, which is based on a homogeneous and isotropic aquifer, is adequate for predicting streamflow in the system under study by Manga, it does not account for heterogeneities that generate preferential flowpaths and, thus, localize spring discharge. We apply a similar model, but attempt to incorporate the effects of high-permeability features on the spring system and identify parameters that contribute to steady spring flow.

Townley (1995) presented a complex algebra-based methodology for obtaining analytical solutions for one-dimensional, periodic ground water flow. His six examples of periodic forcings due to tidal fluctuations, recharge, and pumping apply to one-dimensional and radially symmetric aquifers of finite length, L. Townley's methodology is briefly summarized here.

Flow in a homogeneous aquifer in one dimension (x) is governed by the equation

$$\frac{\partial h}{\partial t} = \frac{T}{S}\frac{\partial^2 h}{\partial x^2} + R \tag{1}$$

where h is the head, t is the time, T is the transmissivity, S is the storativity, and R is the source/sink term.

After Townley (1995), we assume that head and recharge are periodic functions of time. For example, head can be represented by

$$h_{(x,t)} = h_{S(x)} + \mathrm{Re}\{h_{P(x)}\exp(i\omega t)\} \tag{2}$$

where h_S is the steady component of head, h_P is a complex number representing the periodic component of head, $i = \sqrt{-1}$, ω is the frequency of the fluctuations ($2\pi/P$), P is the period of fluctuations, and Re{ } is an operator that takes the real part of a complex number.

Incorporating Equation 2 in terms of both head and recharge into Equation 1 results in the following separate equations for the steady and periodic components of head:

$$T\frac{d^2h_S}{dx^2} + R_S = 0 \tag{3}$$

for the steady component of head and

$$T\frac{d^2h_P}{dx^2} - i\omega S h_P + R_P = 0 \tag{4}$$

for the periodic component of head, where R_S is the steady component of recharge and R_P is a complex number representing the periodic component of recharge. Once Equations 3 and 4 are solved with appropriate boundary conditions, a complete solution for head can be found by incorporating its steady and periodic components into Equation 2 (Townley 1995).

A solution for one-dimensional (horizontal), unconfined ground water flow with steady and periodic recharge (Townley 1995) is applied to the system. The model illustrated and described in Figure 6 is similar to the one presented by Manga (1997) for a homogeneous and isotropic volcanic aquifer. In our system, however, aquifer heterogeneities in the form of high-permeability layers are thought to influence spring flow characteristics. At the watershed scale, models based on an equivalent homogeneous representation of a layered aquifer system employing vertically integrated hydraulic properties will often yield head surfaces and discharge rates that are similar to those of models in which high- and low-permeability strata are represented explicitly (Haitjema 1995). In order to simplify the complex geology, but still incorporate the layered nature of the system into a one-dimensional model, we calculated a total transmissivity (T_{total}) by summing transmissivity estimates for individual hydrostratigraphic units including thin, high-permeability layers as

$$T_{total} = \sum_{i=1}^{n} K_i b_i \tag{5}$$

where K_i is the hydraulic conductivity of an individual layer, n is the number of layers, and b_i is an individual layer thickness. In doing so, we assume that periodic recharge is attenuated by the horizontal resistance of the entire stratified aquifer system.

We assume that flow is primarily in one direction and that the aquifer has a horizontal, impermeable base. There is a ground water divide at $x = 0$ ($dh_S/dx = dh_P/dx = 0$) and a prescribed head ($h_S = h_P = 0$) at $x = L$. Recharge that varies sinusoidally in time is applied to the unconfined aquifer. Because the system is unconfined, the solution utilizes the linearized Boussinesq equation, which is based on the assumption that changes in head are small in comparison to the thickness of the aquifer.

The use of a one-dimensional model, which for an unconfined aquifer is based on the Dupuit-Forcheimer assumptions, for a complex three-dimensional system raises the question of the importance of vertical flow in the system. However, the aquifers in this hydrogeologic setting are thin compared to their lateral extent, and regional numerical modeling by Krohelski et al. (2000) shows that ground water flow is predominantly horizontal within the unlithified aquifer and the upper bedrock aquifers. It should be noted that the solution breaks down where the Dupuit-Forcheimer assumptions break down, that is, where flow lines deviate significantly from horizontal, as in recharge areas or immediately adjacent to discharge areas. For example, this limitation can be important in regions with small recharge areas or where the size of the recharge area is of the same order as the aquifer thickness. However, where the recharge area is sufficiently large, a Dupuit-Forcheimer solution is often adequate (Haitjema 1987).

Using general solutions to Equations 3 and 4,

$$h_S = -\frac{R_S}{2T}x^2 + C_1 x + C_2 \tag{6}$$

and

$$h_P = D_1 \cosh ax + D_2 \sinh ax + \frac{R_P}{i\omega S} \tag{7}$$

where C_1 and C_2 are real constants, D_1 and D_2 are complex constants, and $a = \sqrt{i\omega S/T}$, along with the boundary conditions described previously, the analytical solutions for the

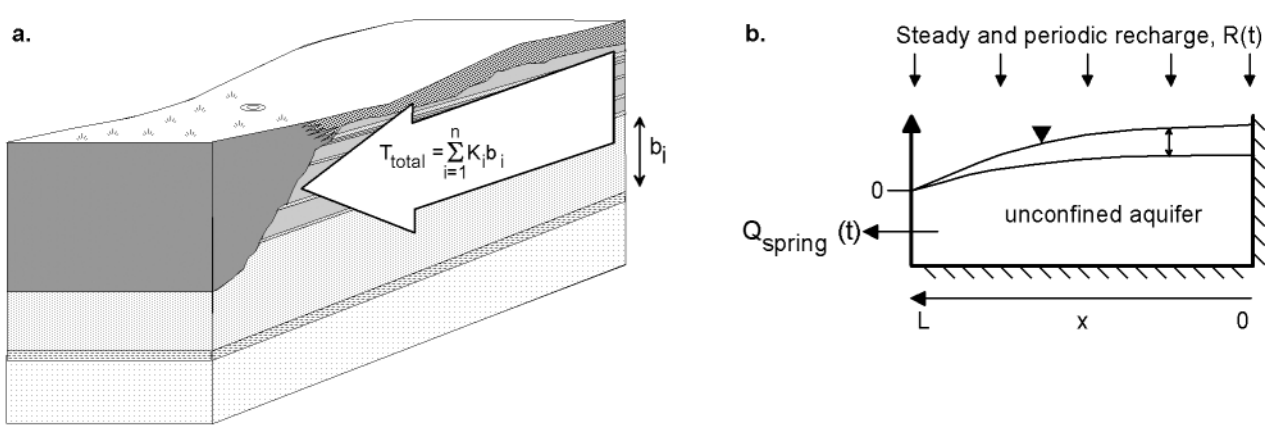

Figure 6. (a) Conceptual model and (b) corresponding sketch of a one-dimensional, unconfined aquifer with vertically integrated hydrogeologic properties.

steady and periodic components of head are (Townley 1995)

$$h_S = \frac{R_S}{2T}(L^2 - x^2) \tag{8}$$

and

$$h_p = \frac{R_p}{i\omega S}\left(1 - \frac{\cosh b\frac{x}{L}}{\cosh b}\right) \tag{9}$$

where

$$b = aL = L\sqrt{i\omega S/T} = \left[2\pi i\left(\frac{L^2S}{TP}\right)\right]^{1/2} \tag{10}$$

After Townley (1995), we present results in terms of a dimensionless ratio, L^2S/TP, on which the solution for the periodic component of head depends (Equation 10). In general, a periodic forcing propagates a greater distance through the one-dimensional aquifer when L^2S/TP is small. The amplitude of the periodic forcing is damped and the phase lag between the periodic forcing and the aquifer response increases as this ratio increases (Townley 1995). It should be noted that these conclusions do not directly apply to a two- or three-dimensional stratified aquifer system, but the qualitative relationships can be assumed to apply (Haitjema 1995).

The net outflow (spring flow) from the aquifer at $x = L$ is also a function of its steady and periodic components:

$$Q_S = -R_S L \tag{11}$$

and

$$Q_P = -R_P L\frac{\tanh b}{b} \tag{12}$$

and, similarly, the periodic component of flow depends on L^2S/TP. Aquifer response at $x = L$ to periodic recharge is illustrated in Figure 7, a plot of dimensionless outflow vs. normalized time for various values of the dimensionless ratio L^2S/TP (Townley 1995).

In order to evaluate the response of an unconfined, homogeneous aquifer to periodic recharge and the impact on spring flow (at $x = L$), values of L^2S/TP were calculated using conditions that might be expected in the Nine Springs area, including

1. A reasonable range of specific yield values (0.15 to 0.2)
2. A period of ~183 d (spring and fall recharge)
3. A reasonable range of aquifer lengths (~1.6 to 6.4 km)
4. A reasonable range of T_{total} estimates (~200 to 420 m^2/d).

The minimum aquifer length (1.6 km) represents the average distance between the six major springs noted earlier and an upgradient surface water divide as shown in Figure 2. However, it is unlikely that the surface watershed for the creek coincides with the ground watershed for the springs, so the maximum aquifer length represents the average distance between a spring and a regional ground water divide based on modeling results of the regional ground water flow system (Krohelski et al. 2000).

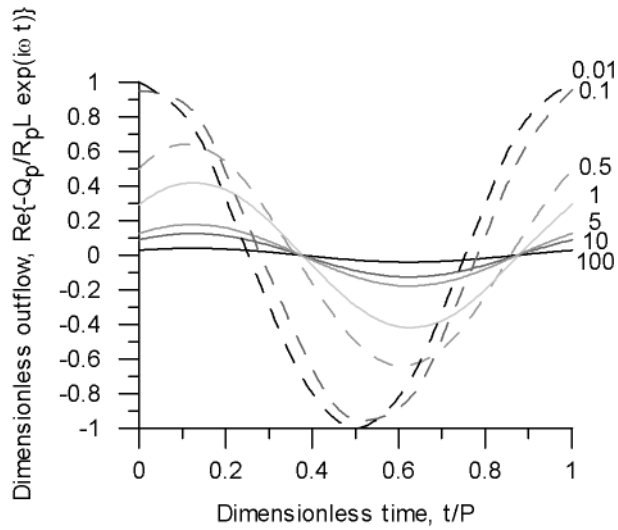

Figure 7. Dimensionless outflow at $x = L$ for values of L^2S/TP (after Townley 1995). The amplitude of the periodic forcing due to recharge decreases and there is a progressive phase lag as L^2S/TP increases. Values of L^2S/TP are listed to the right of lines corresponding to solutions.

The range of T_{total} values was calculated using hydraulic conductivity and thickness estimates for the minimum and maximum number of hydrostratigraphic units, including high-permeability layers, which might be expected in units above the Eau Claire Aquitard. A range of estimates is necessary due to uncertainty in the precise thicknesses of high-permeability features and because bedrock units that are absent near the springs are present with increasing distance from the springs and the buried bedrock valley. Properties of the major bedrock units were estimated based on regional hydrogeologic studies (Bradbury et al. 1999) and thicknesses recorded in well drillers' logs, geologic logs from municipal wells, and geologic logs from monitoring wells drilled as part of this study. Properties of high-permeability layers were based on results of packer tests described previously.

The calculated values of L^2S/TP range from ~5 to > 200. Figure 7 shows that for low values of L^2S/TP, the periodic component of flux propagates into the aquifer and has significant amplitude at the spring outflow. For example, at $L^2S/TP = 0.1$, the amplitude is reduced by < 10% and the phase lag is minor. In contrast, for $L^2S/TP = 1.0$, the amplitude of the flux is reduced by ~60% and there is a phase lag of one-eighth of the period. For $L^2S/TP = 10$, the phase lag is similar, but the amplitude of a periodic flux due to recharge is reduced by almost 90%, resulting in nearly steady flow.

For this system, it is likely that the greatest uncertainties in L^2S/TP are associated with T_{total} and L. However, even using the highest T_{total} estimates and the lowest L estimates, L^2S/TP is > 5. Therefore, the solution, which is based on an equivalent homogeneous representation of a layered aquifer system that includes high-permeability

features, accounts for the observed steady nature of spring flow. Nonetheless, because L is squared in the numerator, uncertainty in this term has an enormous impact on the value of the dimensionless number. The upper limit of L (6.4 km) is constrained by the modeled distance to a regional ground water divide (Krohelski et al. 2000); however, the lower limit (1.6 km) has greater uncertainty. In light of analytical model results, the steady nature of observed spring flow would suggest that the effective aquifer, or flowpath, length is rather long. Numerical modeling provides further constraints on this length, as discussed here.

Numerical Models

Subsurface variations in permeability have long been known to have profound effects on regional ground water flow patterns (Freeze and Witherspoon 1967). With this in mind, the series of numerical simulations tested the conceptual model that attributes observed spring flow characteristics to thin, laterally extensive, high-permeability zones in shallow sandstone aquifers. The modeling procedure consisted of running two classes of transient simulations, those without and those with high-permeability zones. In addition, hydrogeologic parameters were manually adjusted in order to test the sensitivity of spring flow to the hydrogeologic parameter and to ensure that conclusions regarding the effects of high-permeability zones are not limited by initial parameter estimates.

Model Design

The aim was to design numerical ground water flow models that are detailed enough to incorporate the critical features that influence spring flow in the Nine Springs area, but general enough to be representative of other regionally extensive sedimentary aquifers in the Upper Midwest. Therefore, the spring in the model is hypothetical. The models were constructed using the finite-difference code MODFLOW (McDonald and Harbaugh 1988) and the pre- and postprocessor Groundwater Vistas (Rumbaugh and Rumbaugh 1996). The regional hydrostratigraphy, defined by Bradbury et al. (1999), was initially adopted; however, on the basis of the field evidence summarized previously, the upper bedrock aquifer was subdivided into two aquifers that represent (1) the St. Lawrence Formation and the Tunnel City Group, referred to as the Tunnel City Aquifer, and (2) the Wonewoc Formation, referred to as the Wonewoc Aquifer.

The model grid includes 49 columns and 78 rows. The models without high-permeability zones consist of five layers representing four aquifers and one confining unit (Figure 8a). The grid spacing is variable and ranges from 7.6 to 122 m, with finer resolution near a hypothetical spring. The models were designed to represent a slice of a larger ground watershed, so the length of the grid (6.1 km) is much greater than the width (1.5 km). The length is the approximate distance between a spring and a regional ground water divide based on modeling results of the regional ground water flow system (Krohelski et al. 2000). The width is roughly the average distance between major springs in the Nine Springs Wetlands.

The uppermost layer, which represents the unlithified aquifer, is unconfined and nonuniform in thickness. The top elevation is defined by the elevation of the water table. The bottom elevation decreases near the location of a hypothetical spring and the downgradient model boundary in order to represent the sloping wall of a buried valley (Figure 8a). The depth of the bedrock valley was based on auger drilling and vibracore samples in and near the Nine Springs Wetlands (Swanson 2001).

The layer representing the Tunnel City Aquifer is confined. Most of the layer is ~37 m thick, but it thins to 6 m near the downgradient model boundary in order to represent the buried bedrock valley. The bottom three layers, which represent the Wonewoc Aquifer, the Eau Claire Aquitard, and the lower bedrock aquifer, are confined with thicknesses of 43, 4.6, and 183 m, respectively. Thicknesses were based on observed unit thicknesses from DN–1441 and DN–1442 and on thicknesses recorded in geologic logs from nearby municipal wells.

The models that directly tested the effects of high-permeability zones are similar to those without the features, but they include two to four additional layers (Figure 8b). The high-permeability layers were inserted 3 m below the

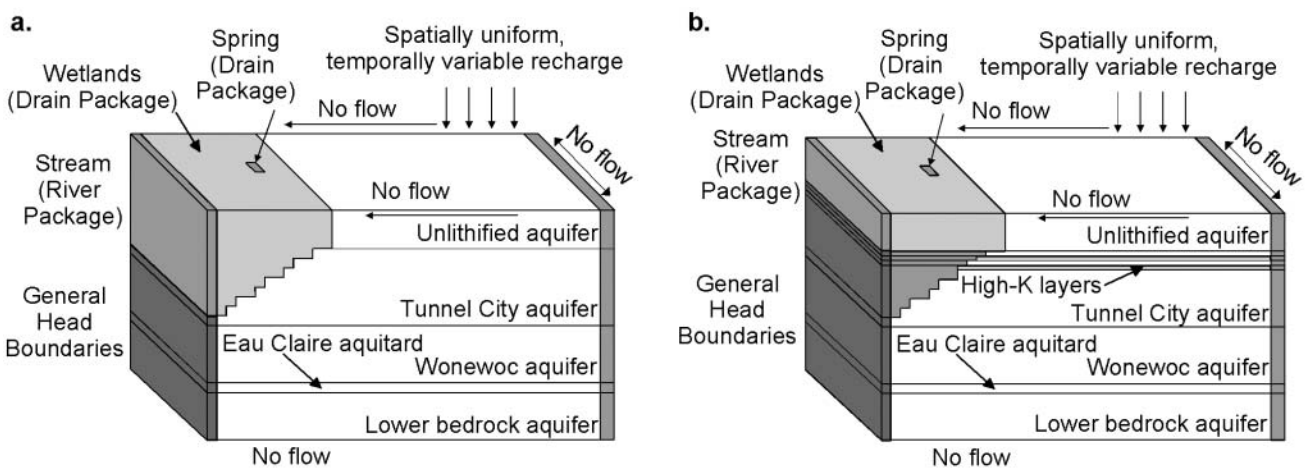

Figure 8. Schematic numerical model grid (a) excluding and (b) including two high-permeability layers. Illustrations are not to scale and the variable grid spacing in the vicinity of the high conductance drain node (spring) is not shown.

top of the layer that represents the Tunnel City Aquifer. For models that include more than one high-permeability layer, the layers are separated by a 1 m thick layer with properties equal to those of the Tunnel City Aquifer.

Boundaries

Boundary conditions are representative of an unstressed ground water flow system. The upgradient external boundary is no-flow and represents a regional ground water divide. At the downgradient external boundary, the River Package (McDonald and Harbaugh 1988) was used to simulate a stream in the uppermost layer. The approximate stage of Nine Springs Creek (260.6 m a.m.s.l.) was used as the river stage, and river conductance was calculated using an estimate of hydraulic conductivity for sandy diamicton (Swanson 1996), a stream width of 3 m, and a streambed thickness of 1.5 m. General head boundaries were assigned to the downgradient boundary nodes in all other layers. The approximate elevation of the lakes in central Dane County (also 260.6 m a.m.s.l.) was used as the external boundary head. The lakes are regional discharge areas; the average distance between Nine Springs Creek and the lakes is 3.2 km. The remaining lateral boundaries, corresponding to flow lines from the ground water divide toward the stream, were defined as no-flow.

The Drain Package (McDonald and Harbaugh 1988) was used to simulate a hypothetical spring and surrounding wetlands. Similar to those in the Nine Springs Wetlands, the hypothetical spring was placed near the margin of the wetlands at the edge of the buried bedrock valley. Drain elevations were based on approximate water table elevations in the vicinity of major springs in the Nine Springs Wetlands (260.6 m a.m.s.l.). The conductance terms for the nodes representing the wetlands are based on (1) hydraulic conductivity estimates for the wetland soils, and (2) grid cell dimensions. The hydraulic conductivity estimates were calculated using typical rates of inflow to a wetland (Hunt 1996) and measured upward vertical gradients (–0.3) in the Nine Springs Wetlands. The conductance term for the hypothetical spring is based on (1) hydraulic conductivity estimates for materials in the immediate vicinity of a spring, and (2) grid cell dimensions. The hydraulic conductivity estimate was calculated using measured spring discharge rates (3 to 9 L/s), approximate spring pool areas (4 to 20 m^2), and ground water gradients (–0.3 to–0.5) in the vicinity of two major springs in the Nine Springs Wetlands.

Areal recharge in the steady-state simulations was applied at a rate of 20.3 cm/yr, which falls within the previously stated range of reasonable values. The rate is an area weighted average recharge based on results of a Precipitation-Runoff Modeling System (Leavesley et al. 1983) surface water model for the nearby Pheasant Branch Watershed, which is in a very similar hydrogeologic setting (Hunt and Steuer 2000; Steuer and Hunt 2001). In the transient simulations, this total annual recharge was apportioned among 12 stress periods of 1 month each in order to simulate major recharge during spring snowmelt and additional recharge during the fall. Recharge was applied at rates of 2.5E–01, 1.7E–01, and 8.1E–02 cm/d during March, April, and May, respectively, and at rates of 8.4E–02 and 8.1E–02 cm/d during October and November. No recharge was applied during the remaining months of a given year; in other words, the steady component of recharge is equal to zero. This temporal distribution corresponds qualitatively to output of a GIS-based, modified Thornthwaite/Mather water-balance model applied to the nearby Pheasant Branch Watershed (Dripps et al. 2000).

Hydraulic Properties

Hydraulic properties were primarily based on the results of several cooperative studies of the regional hydrogeology (Swanson 1996; Bradbury et al. 1999; Krohelski et al. 2000). The uppermost model layer is isotropic, but heterogeneous. Most of the nodes in this layer were assigned a hydraulic conductivity of 5.3E–04 cm/s, representative of sandy diamicton of the Horicon Member of the Holy Hill Formation (Rayne 1993; Swanson 1996). Nodes corresponding to glaciolacustrine deposits in the bedrock valley were assigned a hydraulic conductivity of 2.9E–05 cm/s (Swanson 1996). The layer(s) representing the Tunnel City Aquifer is (are) homogeneous and isotropic and was (were) assigned a hydraulic conductivity value of 1.8E–03 cm/s. This value was assigned on the basis of estimates (Bradbury et al. 1999) and final calibrated values for the upper bedrock aquifer in the Dane County regional ground water flow model (Krohelski et al. 2000). The layers representing the Wonewoc Aquifer and the lower bedrock aquifer are homogeneous and isotropic and were assigned a hydraulic conductivity value of 3.5E–03 cm/s based on estimates by Bradbury et al. (1999) for the lower bedrock aquifer. Hydrogeologic properties of the Wonewoc Aquifer were considered similar to those of the lower bedrock aquifer due to lithological similarities between the two units and results of the slug tests discussed previously. The layer representing the Eau Claire Aquitard is anisotropic (K_h:K_v = 100:1) and was assigned a horizontal hydraulic conductivity value of 1.0E–10 cm/s based on tabulated values of hydraulic conductivity estimates for shale (Freeze and Cherry 1979). Although this is much lower than the value of 2.1E–07 cm/s used to calculate leakance across the Eau Claire Aquitard in the Dane County regional ground water flow model (Krohelski et al. 2000), it is consistent with the observed 18 m head drop across the Eau Claire Aquitard at the monitoring well pair described previously. For the simulations that included one or more high-permeability layers, each layer was assigned horizontal and vertical hydraulic conductivity values of 1.0E–02 cm/s, which was considered a lower limit of the range of reasonable hydraulic conductivity estimates based on the packer test results.

Estimates of storativity are unavailable for the regionally defined hydrostratigraphic units, so values were estimated on the basis of tabulated ranges of specific yield and specific storage for geologic materials (Johnson 1967; Anderson and Woessner 1992). The uppermost model layer was assigned a specific yield of 0.20 and all other layers, excluding the Eau Claire Aquitard, were assigned a specific storage term of 6.9E–05/m. The layer representing the Eau Claire Aquitard was assigned a specific storage term of 9.2E–04/m.

Initial Conditions

Starting heads for the transient simulations were those produced using an average recharge rate (20.3 cm/yr) in the steady-state simulations. Dynamic cyclic conditions were

generated by running the transient models for a number of years employing the temporal recharge pattern specified earlier. Although 5 yr was generally sufficient to produce a repeatable yearly cycle of heads and discharge, each transient model was run for a total of 8 yr.

Modeling Targets

The following targets, representative of conditions in the Nine Springs area, were used to evaluate the two classes of models in terms of their capability to simulate typical spring flow conditions.

1. *A model should simulate a spring discharge rate that is greater than 3 L/s*. This target was based on the observed range of steady discharge rates from individual springs in the Nine Springs Wetlands. Because the model domain represents only a "slice" of a larger ground watershed, the lower limit of observed flow rates was chosen as a discharge target for the hypothetical spring.

2. *A model should simulate steady spring flow under transient conditions*. Due to the precision of the field equipment used in the Nine Springs area, the smallest detectable change in spring flow was ~2% to 5% of the mean flow (Schwar 2002). Changes in spring flow from average flow rates (due to seasonal rainfall and/or snowmelt) for springs in a variety of other hydrogeologic settings range from ~10% to as much as 1000% of the mean flow rate (Atkinson 1977; Bonacci 1995; Manga 1997; Angelini and Dragoni 1997; Mayo and Morris 2000). Therefore, a seasonal response would be expected to result in a change of at least 10% of the mean spring flow rate. For modeling purposes, for a simulated mean spring flow rate > 3 L/s, spring flow was considered steady if seasonal fluctuations from the mean spring flow rate were < 0.3 L/s. For simulated mean spring flow rates of < 3 L/s, spring flow was considered steady if seasonal fluctuations result in changes that were < 10% of the mean flow rate.

3. *A model should simulate diffuse ground water discharge rates to the drain nodes representing wetland areas that are > 0.1 cm/d but < 0.9 cm/d*. This range approximates the limits of literature values for ground water inflow to similar wetland systems (Hunt 1996). The target does not impose a major constraint, but it does provide a guide as to whether typical wetland vegetation could be supported.

4. *A model should produce a reasonable head distribution throughout the model domain*. Areally applied recharge should raise the water table, causing a ground water mound near the upgradient no-flow boundary. The head should decrease near the downgradient boundary, producing a classical parabolic shape (Townley 1995). It can be shown that the degree of ground water mounding generally depends on the recharge rate, the geometry of the system, and the aquifer transmissivity (Haitjema 1995). All simulations used the same recharge rate and the geometry of the system (length, width, overall thickness) did not change among simulations. Therefore, ground water mounding should respond only to changes in the transmissivity of the aquifer system. Results of each simulation were checked to ensure that these changes did not cause unreasonable ground water mounding.

Simulations and Results

Models that include high-permeability zones are more capable of simulating flow conditions that are representative of those in the Nine Springs area. The modeling target of simulating spring flow exceeding 3 L/s was met only in those simulations that included high-permeability layers. The modeling target of simulating steady spring flow was met in all of the simulations. The third modeling target, to simulate diffuse ground water discharge to wetland areas at rates ≤ 0.9 cm/d, was not met in any simulations without high-permeability layers and was met in only some of the simulations that included high-permeability layers. Although simulated rates of diffuse discharge for some simulations with high-permeability layers exceeded the target, ranging up to 1.0 cm/d, these simulations all yielded reduced diffuse discharge compared to models without high-permeability features. The fourth modeling target, to produce a reasonable head distribution throughout the model domain, was met in all of the simulations.

Results are consistent over the ranges of hydrogeologic properties that were used in the sensitivity analysis (Table 1), indicating that conclusions are not limited by initial parameter estimates. For each class of models, simulations tested the sensitivity of spring flow to aquifer properties, such as storativity and hydraulic conductivity. For those models including high-permeability zones, additional simulations tested the sensitivity of spring flow to the number and thickness(es) of high-permeability layer(s) and to leakance and anisotropy of the layers. The results of the sensitivity analysis show that the magnitude of the mean spring flow is sensitive to the thickness, number, and hydraulic conductivity of high-permeability layers and leakage into these layers. Seasonal fluctuations in spring flow are only slightly sensitive to specific yield, hydraulic conductivity, and leakage into the high-permeability layer(s). Table 2 summarizes the results of the sensitivity analysis for each class of numerical models.

The results suggest that preferential pathways of ground water flow correspond to thin, laterally extensive, high-permeability layers in the uppermost bedrock aquifer. The layers influence the magnitude of spring flow because specific discharge is proportional to variations in hydraulic conductivity with depth. The high-permeability layers effectively transmit water to the discharge area and they contribute to the development of artesian flow conditions (Freeze and Witherspoon 1967). In this setting, springs form because the features are truncated by a buried bedrock valley and the overlying unlithified materials are relatively thin.

Because all simulations produced steady spring discharge, they do not provide direct constraints on the steady nature of flow. However, the models do provide constraints on the lengths of flowpaths to the spring. Flow within the high-permeability layer(s) is predominantly horizontal (Figure 9), and the lengths of the flowpaths increase as the number of thin, high-permeability layers increases. Therefore, if such features are indeed present in the Nine Springs system, the numerical modeling suggests that ground water flowpaths to the springs are likely to be long, i.e., closer to the longer values of L used in the previous calculations of L^2S/TP.

Table 1
Summary of Hydrologic Characteristics Used in the Numerical Model Simulations

Hydrologic Characteristic	K (cm/s)		Anisotopry Ratio (K_H:K_V)		S_y or S_s(m)		Number of Layers		Thickness of Layer (m)	
	Range	Base Run	Range	Base Run	Range	Base Run	Range	Base Run	Range	Base Run
Unlithified Aquifer (till, glaciolacustrine)	5.3E-05–5.3E-03, 2.9E-05	5.3E-04, 2.9E-05	—	1:1	0.07–0.30	0.2	—	1	—	variable[a]
Tunnel City Aquifer	8.8E-04–3.5E-03	1.8E-03	—	1:1	6.9E-06–6.9E-04	6.9E-05	1–3	1–2[b]	—	variable[a]
High-Permeability Zones	3.5E-03–1.0E-02	1.0E-02	1:1–6:1[c]	1:1	6.9E-05–6.9E-04	6.9E-05	1–2	1	1–1.5	1.5
Wonewoc Aquifer	1.8E-03–7.1E-03	3.5E-03	—	1:1	6.9E-06–6.9E-04	6.9E-05	—	1	—	43
Eau Claire Aquitard	1.0E-10–1.0E-09	1.0E-10	—	100:1	9.2E-05–9.2E-03	9.2E-04	—	1	—	4.6
Lower Bedrock Aquifer	1.8E-03–3.5E-03	3.5E-03	—	1:1	6.9E-06–6.9E-04	6.9E-05	—	1	—	183

— Parameter was not varied
[a]The thickness of the layer is variable due to the buried bedrock valley, but the overall thickness is consistent among simulations.
[b]One layer for the base run without high-permeability layers and two layers for the base run with high-permeability layers.
[c]The 6:1 anisotropy ratio assumes K_H = 1.0E-02 cm/s and K_V = 1.8E-03 cm/s.

Discussion and Conclusions

Results of both the analytical and numerical models support the hypothesis that springs with steady flow rates can be generated by preferential flow along high-permeability features in a shallow sandstone aquifer. The analytical model demonstrates that it is unlikely that a periodic forcing due to seasonal variations in areal recharge would propagate to springs in a setting where high-permeability features exist. The numerical models demonstrate that high-permeability features can also influence the magnitude of spring flow.

Table 2
Results of Sensitivity Analysis for the Numerical Models

Aquifer and Hydrologic Characteristic Varied[a]	Hydrologic Value Simulated	Mean Spring Flow (L/s)	Minimum Spring Flow (L/s)	Maximum Spring Flow (L/s)
Unconsolidated Aquifer				
specific yield	0.3	7.2E-01	7.1E-01	7.3E-01
	0.07	7.3E-01	6.9E-01	7.6E-01
hydraulic conductivity (cm/s)	5.3E-03	7.2E-01	7.0E-01	7.4E-01
	5.3E-05	7.3E-01	7.1E-01	7.5E-01
Tunnel City Aquifer				
hydraulic conductivity (cm/s)	3.5E-03	1.3E+00	1.2E+00	1.3E+00
	8.8E-04	4.3E-01	4.2E-01	4.4E-01
Wonewoc Aquifer				
hydraulic conductivity (cm/s)	1.8E-03	8.0E-01	7.8E-01	8.1E-01
	5.3E-03	6.7E-01	6.5E-01	6.9E-01
	7.7E-03	6.3E-01	6.1E-01	6.4E-01
Base run without high-permeability layer		7.2E-01	7.1E-01	7.4E-01
Unconsolidated Aquifer				
specific yield	0.1[b]	4.1E+00	3.9E+00	4.2E+00
	0.3[c]	5.1E+00	5.1E+00	5.2E+00
	0.1[c]	5.1E+00	5.0E+00	5.3E+00
High-Permeability Zones				
hydraulic conductivity (cm/s)	3.5E-03[d]	2.7E+00	2.6E+00	2.7E+00
anisotropy	6:1[b]	3.5E+00	3.4E+00	3.5E+00
	6:1[c]	3.7E+00	3.6E+00	3.7E+00
thickness (m)	1	4.0E+00	4.0E+00	4.1E+00
number	2	5.1E+00	5.0E+00	5.2E+00
Base run with one 1.5 m thick high-permeability layer		4.6E+00	4.5E+00	4.7E+00

[a]Only those simulations with changes from base runs to the mean, minimum, and maximum spring flow are reported. See Table 1 for the full range of values used in the sensitivity analysis.
[b]Change applies to model with one 1 m-thick high-permeability layer.
[c]Change applies to model with two high-permeability layers.
[d]Change applies to base run.

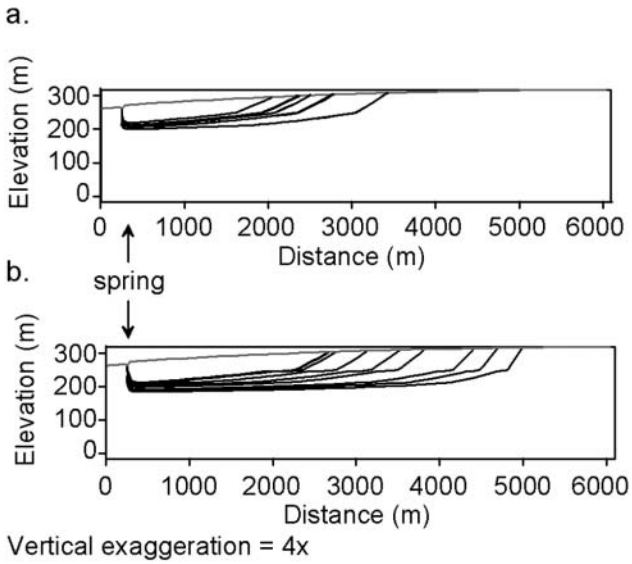

Figure 9. Numerical model grid along column 24 from simulations that (a) exclude and (b) include high-permeability layers. The lengths of the flowpaths to the springs, shown by particle traces generated by reverse particle tracking using MODPATH (Pollock 1994), increase when high-permeability layers are explicitly modeled.

The analytical model identifies parameters to which steady vs. periodic spring flow is most sensitive. The magnitude of L^2S/TP, and the associated damping of spring flow variations, is highly sensitive to the aquifer length, L, along a flowpath or between major boundaries. The true lengths of the ground water flowpaths to the major springs in the Nine Springs system are unknown. However, the numerical models show that not only does the magnitude of spring flow increase when thin, high-permeability layers are explicitly modeled, but that the lengths of flowpaths to the simulated spring also increase (Figure 9).

The numerical simulations demonstrate that the magnitude of spring flow is sensitive to the number, thickness, and hydraulic conductivity of the layer(s), all of which affect the transmissivity of the layers and ultimately the distribution of specific discharge with depth. However, all of the simulations produced steady spring flow regardless of these changes in transmissivity or whether high-permeability layers were explicitly modeled. Therefore, we conclude that while explicitly modeling thin, high-permeability layers increases the magnitude of spring flow, their explicit representation apparently does not increase the effective transmissivity, T_{total}, for use in the analytical solution, enough to cause significant changes in L^2S/TP. Alternatively, increases in T_{total} are offset by the accompanied increase in L.

While the stratified aquifer system under study is actually composed of an unconfined unlithified aquifer overlying a confined system, most of the "lumped" system's capacity for storage is due to filling and draining of pores near the water table (quantified by specific yield) rather than from elastic storage that contributes to confined aquifer storativities, which are commonly several orders of magnitude smaller than specific yield (Freeze and Cherry 1979). Therefore, in this setting, the response of spring flow to periodic recharge is relatively insensitive to storativity because estimates of specific yield are not likely to differ by more than a factor of two.

Although aquifer dimensions and properties are not likely to vary significantly through time, recharge may occur more or less frequently than during two seasons, spring and fall. Increasing the frequency (reducing the period) of the periodic forcing increases the magnitude of L^2S/TP. Therefore, conclusions regarding the steady nature of spring flow would not change if the frequency of ground water recharge increased. However, under drought conditions, the frequency of recharge decreases, which may result in measurable, but still damped, fluctuations in spring flow that lag behind the fluctuations in recharge.

Finally, it should be noted that what is considered steady or periodic flow depends on the precision of equipment used to measure outflow, as well as on damping of the recharge signal. For example, variations in outflow of 10% might be measurable in a large spring, but not in a small one, or a particularly large recharge event might be measurable even though it is damped 90%.

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