Karst Spring Responses Examined by Process-Based Modeling

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Abstract

Ground water in karst terrains is highly vulnerable to contamination due to the rapid transport of contaminants through the highly conductive conduit system. For contamination risk assessment purposes, information about hydraulic and geometric characteristics of the conduits and their hydraulic interaction with the fissured porous rock is an important prerequisite. The relationship between aquifer characteristics and short-term responses to recharge events of both spring discharge and physicochemical parameters of the discharged water was examined using a process-based flow and transport model. In the respective software, a pipe-network model, representing fast conduit flow, is coupled to MODFLOW, which simulates flow in the fissured porous rock. This hybrid flow model was extended to include modules simulating heat and reactive solute transport in conduits. The application of this modeling tool demonstrates that variations of physicochemical parameters, such as solute concentration and water temperature, depend to a large extent on the intensity and duration of recharge events and provide information about the structure and geometry of the conduit system as well as about the interaction between conduits and fissured porous rock. Moreover, the responses of solute concentration and temperature of spring discharge appear to reflect different processes, thus complementing each other in the aquifer characterization.

Introduction

Drinking water supply in karst areas often depends on ground water resources as surface water is usually scarce. Unfortunately, karst ground water resources are highly vulnerable to contamination due to the rapid spreading of pollutants in solution conduits. Effective strategies for both management and protection of ground water resources must be based on reliable information about the location, geometry, and hydraulic properties of the karst conduit system. Karst springs that are supplied by well-developed conduit systems show rapid responses to recharge events of both spring discharge and physicochemical parameters of the discharged water. Thus, the evaluation of time series of these parameters has been proposed, for instance, by Ashton (1966), as a method for the characterization of karst conduit systems.

This work intends to investigate how far spring responses reflect geometric properties of the conduit system. To this end, a process-based modeling tool is applied to simulate flow and transport in hypothetical but nevertheless realistic karst systems. Spring responses predicted by numerical simulations are evaluated with regard to conduit geometry, and results are compared to the actual properties of the known modeled conduit system.

Method

The heterogeneity of karst aquifers can be described conceptually as a dual flow system that comprises two components, “diffuse flow” in the fissured porous rock and “localized conduit flow” (Atkinson 1977). Liedl et al.
(2003) present a modeling tool accounting for both flow components by coupling a discrete pipe network, representing the conduit system, to MODFLOW-96 (Harbaugh and McDonald 1996), which simulates Darcian flow in the fissured porous rock.

Flow calculation in the discrete pipe network is based on Kirchhoff’s law, which states that total inflow and outflow balance at each node $i$:

$$\sum_{j} Q_{ij} + R_{Ci} + q_{ex,i} = 0$$

(1)

where $Q_{ij}$ is discharge in pipe $j$ connected to the node $i$ (m$^3$/s), $R_{Ci}$ is point recharge (m$^3$/s), and $q_{ex,i}$ is flow from the fissured system into the conduit system at node $i$ (m$^3$/s). Flow in each pipe of the network is governed by the Darcy-Weisbach equation:

$$\Delta h_C = \frac{\lambda L}{2 ga} v^2$$

(2)

where $\Delta h_C$ is head difference along a pipe (m), $\lambda$ is friction factor, $a$ is pipe diameter (m), $L$ is length of the pipe (m), $v$ is average velocity of flow through the pipe (m/s), and $g$ is gravitational acceleration (m/s$^2$). In the case of laminar flow, the Hagen-Poiseuille formula is applied to determine the friction factor. At higher Reynolds numbers, the implicit Colebrook-White formula is used.

The discrete pipe-network model described previously is coupled to the fissured system, i.e., to MODFLOW, at the nodes of the pipe network by a linear exchange term (Barenblatt et al. 1960):

$$q_{ex,i} = \gamma (h_{Fi} - h_{Ci})$$

(3)

where $h_{Ci}$ and $h_{Fi}$ denote hydraulic heads (m) at the node $i$ in the discrete pipe network and in the continuum model MODFLOW, respectively, and $\gamma$ is the exchange coefficient (m$^2$/s), which is dependent on the hydraulic conductivity of the fissured porous rock, the surface area available for flow between the two flow systems, and a factor determined by the geometry of the conduits and the interconduit blocks (Bauer et al. 2003).

This continuum-pipe flow model was initially designed for the simulation of conduit development in karst aquifers (Bauer et al. 2003; Birk et al. 2003; Liedl et al. 2003) and thus termed Carbonate Aquifer Void Evolution (CAVE). In this work, CAVE is used to simulate responses of spring discharge to recharge events. In order to allow the prediction of physicochemical responses to recharge events, modules simulating solute and heat transport in conduits were added to CAVE.

The simulation of reactive solute transport in conduits is based on the one-dimensional advection-dispersion equation to which a source term has been added that accounts for the increase of solute concentration due to the dissolution of rock:

$$\frac{\partial c}{\partial t} = -v \frac{\partial c}{\partial z} + D \frac{\partial^2 c}{\partial z^2} + \frac{4}{a} F$$

(4)

where $c$ denotes solute concentration (mol/m$^3$), $v$ flow velocity (m/s), $D$ dispersion coefficient (m$^2$/s), $a$ conduit diameter (m), $F$ dissolution rate (mol/m$^2$/s), $t$ time (s), and $z$ spatial coordinate along the conduit (m). As flow in conduits is typically turbulent, the dispersion coefficient is calculated using relationships for turbulent pipe flow established by Taylor (1954). Birk et al. (2005) provide details on this approach and describe its application to tracer-test modeling. The coefficient $4/a$ in Equation 4 represents the ratio of surface area and volume of the conduit, which is required to transform the dissolution rate $F$ into a change in concentration. The dissolution rate $F$ is calculated by (Svensson and Dreybrodt 1992; Dreybrodt et al. 1996; Liu and Dreybrodt 1997; Eisenlohr et al. 1999):

$$F = k \left(1 - \frac{c}{c_{eq}}\right)^n$$

(5)

where $c_{eq}$ is the equilibrium concentration with respect to the dissolved mineral (mol/m$^3$). The rate constant $k$ (mol/m$^2$/s) is generally dependent on factors such as type of rock, flow conditions, etc. The exponent $n$ (-) equals unity if the dissolution process is diffusion controlled, i.e., if the rate-limiting step is the diffusion of the dissolved species from the conduit wall into the mobile conduit water. If the dissolution rate is limited by the surface reaction, $n$ is a positive number that has to be determined experimentally. Usually, a switch from $n = 1$ to $n > 1$ is observed at high relative saturation states. In this work, a typical rate law with a kinetic switch from first order ($n = 1$, $k = 4 \times 10^{-7}$ mol/m$^2$/s) to higher order ($n = 4$, $k = 4 \times 10^{-4}$ mol/m$^2$/s at $c = 0.9c_{eq}$ is used.

In addition to the solute transport module, the model includes a heat transport module simulating heat convection in the conduit water and heat conduction in the adjacent rock. The heat transport simulation is based on the equation of heat convection expanded by a source term accounting for heat transfer between the conduit water and the conduit wall (Birk et al. 2001; Birk 2002, 17–18):

$$\frac{\partial T}{\partial t} = -v \frac{\partial T}{\partial z} + \frac{h}{c_w \rho_w} \frac{4}{a} (T_s - T)$$

(6)

where $h$ denotes heat transfer coefficient (J/m$^2$/s/K), $T$ temperature of the conduit water (K or °C), $T_s$ temperature at the conduit wall (K or °C), $c_w$ specific heat of water (J/kg/K), and $\rho_w$ water density (kg/m$^3$). To calculate the temperature at the conduit wall, the equation of heat conduction in cylindrical coordinates is solved (Carslaw and Jaeger 1959, 188):

$$\frac{\partial T_r}{\partial t} = \kappa_r \left(\frac{\partial^2 T_r}{\partial r^2} + \frac{1}{r} \frac{\partial T_r}{\partial r} \right)$$

(7)

where $T_r$ denotes the rock temperature (K or °C), $\kappa_r$ the thermal diffusivity of the rock (m$^2$/s), and $r$ the radial coordinate (m).

Heat and solute transport equations are solved using explicit finite differences. Concentration and temperature of inflow to each pipe are obtained by assuming an instantaneous mixing of water at the conduit intersections. Properties of both point recharge into conduits and flow from the fissured system into conduits are specified as boundary conditions. In order to solve Equation 7, a fixed rock temperature is specified as boundary...
condition at a radial distance large enough to assure that heat flux across this boundary is negligible.

The modeling tool outlined previously is used to simulate flow and transport in a hypothetical karst catchment. The spring responses resulting from these simulations are evaluated, focusing on an approach for the estimation of the volume of the karst conduit system that was first proposed by Ashton (1966) and later applied by, e.g., Atkinson (1977) and Ryan and Meiman (1996). This approach makes use of the time lag between the response of spring discharge and that of physicochemical parameters of the discharge. The discharge generally responds a lot quicker to recharge events than the physicochemical parameters, such as solute concentration or water temperature, because the increase in hydraulic pressure is almost instantaneously transmitted through water-filled (phreatic) conduits to the spring, while significant changes in fluid properties occur only after the newly recharged water arrives at the spring (Figure 1). Ashton (1966) suggested that the water volume discharged during this time lag had been displaced from the conduit storage and thus provides an estimate for the volume of the (phreatic) karst conduit system. Time lags between hydraulic and physicochemical spring responses resulting from the model scenarios described subsequently were identified using “those points where the graph undergoes a rapid change” (Ashton 1966), i.e., the onset of the parameter change rather than the peak time of the respective parameter. Using the peak might often produce similar results but runs into difficulties if the discharge only slowly approaches its maximum value and thus the peak is not well defined.

A single cylindrical conduit (length: 1200 m) embedded in fissured porous limestone (hydraulic conductivity, $10^{-2}$ m/s; effective porosity, 0.01) drains the hypothetical karst catchment considered in this study (Figure 2). Several model scenarios are considered, in which the conduit is either constant in diameter (0.5 m) or composed of two cylindrical segments of equal length with diameters of 0.35 and 0.61 m, but the total conduit volume is always kept constant at 236 m$^3$. A fixed-head boundary condition (1 m) is prescribed at the left-hand side of the model domain (1750 m by 750 m), and uniformly distributed recharge ($10^{-8}$ m/s, i.e., 316 mm/year) infiltrates into the fissured porous rock. Figure 2 shows the resulting steady-state flow field prior to the injection of point recharge. The conduit focuses most of the flow to a single point, i.e., the karst spring, while only a small portion discharges as diffuse seepage from the fissured porous rock. Both rate and duration of the point recharge are varied in a number of model runs. Calcium concentrations are assumed to be zero for point recharge and 2 mmol/L for flow from the fissured porous rock. For heat transport simulations, a temperature of point recharge of 6°C and a pre-event aquifer temperature of 8°C have been used.

Results

Figure 3 depicts the responses of spring discharge and solute concentration in the spring water resulting from point recharge of 5 L/s infiltrating into a constant-diameter conduit embedded in fissured porous limestone over a time period of 1 h. For comparison, the corresponding concentration response of an isolated conduit that is hydraulically decoupled from the fissured porous rock is shown. Clearly, the time lag between the rise in spring discharge and the subsequent drop in concentration can be much greater in a dual flow system than in an isolated conduit. The water volume discharged during the
time lag comprises both preevent conduit water and water derived from the fissured porous rock. Hence, the method suggested by Ashton (1966) tends to overestimate the volume of the conduit system because it disregards the contribution of the fissured system. In this example, the error amounts to ~50%, i.e., the water volume discharged during the time lag between rise in spring discharge and drop of concentration exceeds the actual conduit volume by a factor of 1.5. A straightforward method for the quantitative determination of the fissured-system contribution, which could be used to correct the volume estimate, is not available though.

Figure 4 summarizes volume estimates obtained for a number of simulations in which both the rate and duration of point recharge have been varied. Generally, the error decreases with increasing intensity of recharge pulses. Thus, it can be recommended to use the most intense recharge events for the estimation of conduit volumes. Increasing duration of recharge pulses also leads to more accurate volume estimates as long as the recharge period is significantly shorter than the time lag. The volume estimate, however, is insensitive to further increases in pulse duration if the recharge period exceeds the duration of the time lag.

Figure 4 also compares volume estimates obtained for the constant-diameter conduit to those obtained for conduits that have identical volumes but are composed of two segments different in diameter. It turns out that the volume estimate is most accurate if the large-diameter conduit is located at the spring. In this case, much of the flow contributed by the fissured porous rock enters the conduit close to the spring, while a relatively small proportion of this flow component is mixed with the conduit water in the narrow conduit passage connected to the sink. Therefore, the point recharge component at the spring is less delayed and, consequently, the time lag more representative of the actual conduit volume as compared to the case where much of the flow from the fissured porous rock is mixed to the conduit water at larger distance from the spring. It should also be noted that the sensitivity of volume estimates on recharge conditions appears to depend on the conduit geometry. This suggests that the dependency of volume estimates on recharge conditions can potentially be evaluated to infer information about the conduit geometry, especially about the relative distribution of the conduit volume within the karst catchment. Although this concept is not easily implemented in a straightforward approach for the aquifer characterization, it could be used for calibration purposes in the field application of process-based models, such as the one considered here.

The temperature of spring discharge also represents a physicochemical parameter that can potentially be used to estimate conduit volumes. To compare responses of temperature and solute concentration, heat transport was simulated in the model scenario considered previously (Figures 2 and 3). Figure 5 reveals that the temperature response is significantly dampened and appears to be delayed compared to the concentration response. These effects are caused by thermal interaction between the conduit water and the adjacent rock. As heat transfer from the conduit wall into the conduit water is fast, water infiltrating the aquifer rapidly adjusts to the rock temperature. At the same time, however, conduit walls cool down because heat conduction within the rock is relatively slow. Hence, the first recharge water arriving at the spring has adjusted to temperature values close to the preevent aquifer temperature, while ongoing infiltration causes continuing cooling of conduit walls and consequently decreasing water temperatures (Birk 2002, 104). Thus, water temperature represents a highly “reactive” tracer, which is less suitable than solute concentrations for the estimation of conduit volumes with Ashton’s (1966) approach. Yet, the field application of a numerical heat transport model presented by Liedl et al. (1998) demonstrates that reasonable estimates of both conduit volumes and surface areas of rock-water interfaces can be obtained from inverse

Figure 4. Volume estimates based on time lags between rise of spring discharge and drop of concentration evaluated by simulating solute transport in a single conduit embedded in fissured porous rock. Actual conduit volumes were identical in all simulations, while the conduit was either constant in diameter (0.5 m; solid lines) or composed of two segments of equal length with diameters of 0.35 and 0.61 m (dashed lines, narrow passage connected to the sink; dotted lines, narrow passage connected to the spring). The duration of recharge pulses was 1 h (×), 5 h (+), and 10 h (○).

Figure 5. Comparison of temperature (solid curve) and concentration (dashed curve) responses. Temperature and concentration axes have been scaled to represent relative parameter changes equivalently.

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modeling of spring water temperatures. This suggests that both parameters can independently be used to infer properties of the conduit system, thus complementing each other in the aquifer characterization.

Conclusions

Process-based modeling reveals that the conduit volume estimated from the time lag between hydraulic and physicochemical spring responses tends to be higher than the actual conduit volume in dual karst flow systems. The time lag method can provide reasonable order-of-magnitude estimates though and appears to be well suited, provided conduit flow dominates. Results from the model scenario considered here also suggest that the accuracy of the volume estimate and its sensitivity to recharge conditions depend on the conduit geometry. Thus, the sensitivity to recharge conditions can potentially be used in a characterization approach based on inverse modeling of hydraulic and physicochemical spring responses. Combining the evaluation of solute concentration and temperature of spring discharge may further reduce the ambiguity of the model calibration in that type of approach as the responses of both parameters appear to reflect different transport processes. Future research will have to employ the methods introduced here to examine more complex types of setting, e.g., with conduit systems of irregular geometry, with both distributed and point recharge pulses, etc. It is anticipated though that the general conclusions given previously also apply to more complex settings.

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