A new distance-drawdown method for aquifers with anisotropy on the horizontal plane is presented. The method uses scalar transformation to convert to an equivalent, isotropic medium, thus permitting application of the Cooper-Jacob Method. The method is applicable to cases where at least one ellipse of equal drawdown can be delineated but can also be applied where no ellipse can be discerned from the data. In the latter case, a least-squares regression approach can be employed to estimate the orientation and magnitude of the anisotropy. The regression $R^2$ value provides a quantitative assessment of the degree to which the drawdown data are indicative of a systematic areal anisotropy in the aquifer or whether the data simply reflect natural aquifer heterogeneity. In addition to confined aquifers, this methodology, like the Cooper-Jacob Method, is also applicable to unconfined aquifers either before the onset of delayed drainage or following the completion of delayed drainage provided that the $\alpha$ value meets the recommended criterion.
Transformation of Anisotropic Systems

In anisotropic aquifers, ground water flow lines are generally not perpendicular to equipotential lines, making definition of ground water flow paths considerably more difficult. However, the analysis can be facilitated by employing a simple transformation technique where the coordinates in an anisotropic aquifer are converted to those in an isotropic aquifer (Cedergren 1977; Freeze and Cherry 1979). In a homogeneous aquifer with anisotropy in the horizontal plane and with principal hydraulic conductivities $K_X$ and $K_Y$, the hydraulic conductivity ellipse has semiaxes of $\sqrt{K_X}$ and $\sqrt{K_Y}$ (Figure 1).

The transformation to an isotropic medium traditionally is accomplished in either of two ways:

1. Expand the scale of the region of flow in the $K_Y$ direction by multiplying by the factor $\sqrt{K_X/K_Y}$.
2. Reduce the scale of the region of flow in the $K_X$ direction by dividing by the factor $\sqrt{K_X/K_Y}$.

Expanding the scale in the $K_Y$ direction produces the larger, solid line circle of radius $\sqrt{K_X}$ (Figure 1). In this expanded circle, the medium can be treated as isotropic with a hydraulic conductivity of $K_X$. Reducing the scale in the $K_X$ direction produces the smaller, solid line circle of radius $\sqrt{K_Y}$ (Figure 1). In this smaller circle, the medium can be treated as isotropic with a hydraulic conductivity of $K_Y$.

For aquifer test analysis of aquifers anisotropic on the horizontal plane, the transformed system must be isotropic (as are both transformed systems in Figure 1) and the resultant hydraulic conductivity of the transformed system must also equal the equivalent hydraulic conductivity ($K_E$) or $\sqrt{K_XK_Y}$. This objective can be achieved by an equally valid, but slightly different, transformation approach, as follows.

1. Reduce the scale of the region of flow in the $K_X$ direction by dividing by the fourth root of the anisotropy $\sqrt{K_X/K_Y}$.
2. Expand the scale of the region of flow in the $K_Y$ direction by multiplying by the fourth root of the anisotropy $\sqrt{K_X/K_Y}$.

The circle produced by this transformation (the dashed circle in Figure 1) allows for the geologic medium to be treated as isotropic with a hydraulic conductivity of $\sqrt{K_XK_Y}$, the equivalent hydraulic conductivity.

A well pumping from an aquifer possessing anisotropy on the horizontal plane produces an elliptical cone of influence with contours of equal drawdown forming ellipses having the same ratio of major and minor axes as the hydraulic conductivity ellipse for the aquifer. Consequently, the alternative transformation technique described previously will accurately transform the concentric ellipses of drawdown associated with an anisotropic medium to the concentric circles of drawdown expected in an isotropic medium of equivalent hydraulic conductivity and identical storativity and permits analysis of distance-drawdown data in an aquifer with anisotropy on the horizontal plane using the Cooper-Jacob Method.

Methodology

In applying this method to an aquifer with anisotropy on the horizontal plane, there are three possible conditions.

1. There may be a sufficient number of suitably located observation wells so that at least one ellipse of equal drawdown can be defined by the data (case 1).
2. There may be a sufficient number of observation wells to define the major axis of the areal anisotropy but not the full dimensions of any single drawdown ellipse and, therefore, not the anisotropy ratio (case 2).
3. There are not sufficient observation wells to define either the major axis or the full dimensions of any single drawdown ellipse (case 3).

When there are sufficient aquifer test observation wells to define or approximate at least one elliptical contour of equal drawdown, the method involves the following sequential steps:

1. Fit an ellipse of equal drawdown to the data.
2. Measure the length of the major semiaxis, $a$, and the minor semiaxis, $b$, of the ellipse. Let the $x$-axis be parallel to the ellipse’s major axis and the $y$-axis be parallel to the ellipse’s minor axis.
3. Calculate $ab$, which equals $\sqrt{K_X/K_Y}$.
4. Reduce the scale of the region of flow in the $K_X$ direction by dividing by the factor $\sqrt{K_X/K_Y}$.
5. Expand the scale of the region of flow in the $K_Y$ direction by multiplying by the factor $\sqrt{K_X/K_Y}$.
6. Calculate or measure the transformed distance between the pumping well and each of the observation wells.

Figure 1. Hydraulic conductivity ellipse for an anisotropic aquifer with $K_X/K_Y = 5$. The circles represent different isotropic transformations (adapted from Freeze and Cherry 1979).
7. Plot the observed drawdowns vs. transformed distances from the pumping well on semilogarithmic paper with drawdown on the Cartesian scale and distance on the logarithmic scale.

8. Calculate the equivalent aquifer transmissivity ($T_E$) and storativity ($S$) of the aquifer using the Cooper-Jacob "straight-line," distance-drawdown method. Confirm that the "$u$" value for all data points is <0.01 as required by the methodology. In practice, values as high as 0.10 will still yield useful results for most applications.

9. Calculate $T_X$: $T_X = T_E \sqrt{K_X/K_Y}$ or $T_X = T_E (a/b)$

10. Calculate $T_Y$: $T_Y = T_E / \sqrt{K_X/K_Y}$ or $T_Y = T_E (b/a)$

As an example, consider drawdowns (Figure 2) calculated using the method of Hantush and Thomas (1966) with $T_E = 99.4 \text{ m}^2/\text{d}$, $T_X = 347.9 \text{ m}^2/\text{d}$, $T_Y = 28.4 \text{ m}^2/\text{d}$, $S = 0.0001$, $T_X/T_Y = 12.25$, $Q = 163.6 \text{ m}^3/\text{d}$, and $t = 24 \text{ h}$. Following steps 4 to 6 of the methodology, the transformed coordinates and transformed radial distance to each observation well are calculated. The transformed radial distances $r_T$ are then plotted vs. drawdown in accordance with step 7 and will plot as a straight line on semilogarithmic paper (Figure 3). In contrast, drawdowns plotted vs. the untransformed (actual) distance from the pumping well do not form a straight line but show considerable scatter owing to the anisotropy of the aquifer and the associated elliptical cone of influence (Figure 3).

The transformed data can now be analyzed using the Cooper-Jacob distance-drawdown method as presented in Figure 3. The transmissivities along the major and minor axes of the hydraulic conductivity ellipse are then calculated as follows:

$$T_X = T_E (a/b)$$

$$T_X = 99.8 \text{ m}^2/\text{d} \times (3.5)$$

$$T_X = 349.3 \text{ m}^2/\text{d}$$

$T_Y = T_E (b/a)$

$T_Y = 99.8 \text{ m}^2/\text{d} / (0.286)$

$T_Y = 28.5 \text{ m}^2/\text{d}$

The method recovers the initial assumed values of transmissivity and storativity (Figure 3) within the limits of the precision of measuring $\Delta S_{LC}$ and $r_0$.

If sufficient wells exist to estimate the orientation of the major axis of anisotropy, but not the full dimensions of any single drawdown ellipse (case 2), the magnitude of the anisotropy that best fits the data points can be determined as follows:

1. Choose a range of probable areal anisotropies that may fit the data.
2. For each anisotropy, transform the $x$ and $y$ coordinates of the well locations as specified in steps 4 and 5 of the case 1 procedure.
3. Recalculate the transformed radial distance, $r_T$, of each observation well as specified in step 6 of the case 1 procedure.

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1It should be noted that the values of $u$ are <0.01 for all data points.

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**Figure 2.** Plan view of aquifer test site showing observation wells and the 0.75-m ellipse of equal drawdown.

**Figure 3.** Distance-drawdown analysis using the Cooper-Jacob method.1
4. Plot the observed drawdowns vs. transformed distances from the pumping well on semilogarithmic paper with drawdown on the Cartesian scale and distance on the logarithmic scale.
5. Fit a straight line to the data and calculate a least-squares residual, $R^2$.
6. Repeat steps 2 to 5 for each candidate value of anisotropy, generating a value of $R^2$ for each anisotropy. The anisotropy that best fits the data is indicated by the maximum calculated value of $R^2$.

The plot of $R^2$ vs. anisotropy for a range of anisotropies using the data from case 1 (Figure 4) indicates a distinct peak or maximum at the anisotropy that best fits the data, in this case, 12.25, which agrees with the originally assumed value. Steps 4 through 10 of the basic methodology can now be implemented to calculate $T_E$, $T_X$, $T_V$, and storativity. Because in this instance the drawdowns were calculated from assumed (homogeneous) aquifer properties rather than measured in the field, the $R^2$ value for this anisotropy is 1.000. In practice, lower values of $R^2$ should be expected, reflecting the natural heterogeneities of aquifers. The magnitude of the maximum $R^2$ value provides a quantitative assessment of the degree to which the aquifer possesses a systematic anisotropy. A poor fit may indicate that the aquifer is simply heterogeneous, rather than anisotropic. This optimization approach may also be used in case 1 to test whether a different anisotropy might produce a better fit to the data than the eye, alone, can discern.

If the number or location of observation wells and the associated drawdown data do not seemingly reveal any anisotropy-related pattern in the drawdown data (case 3), the x-axis can be rotated about the pumping well at several different angles or at regular angle intervals and the previous optimization approach applied at each interval. While analytically laborious, this method will ultimately yield the maximum $R^2$ value, thereby defining the orientation and anisotropy that best fits the data. It may also suggest that no anisotropy is indicated in the data.

Conclusions

A method for distance-drawdown analysis of aquifer test data for aquifers possessing anisotropy on the horizontal plane was developed in which the drawdown data are transformed to permit application of the Cooper-Jacob method (1946) by using a scalar transformation of the aquifer coordinate system to that of an equivalent isotropic medium with a transmissivity equal to the aquifer’s equivalent transmissivity. The method’s validity was demonstrated by its ability to recover initial values of transmissivity and storativity used to generate simulated drawdown data. As with the Cooper-Jacob method, this methodology is particularly applicable to confined aquifers but can also be applied to unconfined aquifers either before the onset of delayed drainage or following the completion of delayed drainage, provided that the $u$ value meets the recommended criterion. In cases where the limited number of observation wells does not permit identification of any single ellipse of equal drawdown, the method allows identification of the orientation and magnitude of the anisotropy by means of a simple regression analysis using all available observation well data. The method is believed to be a useful adjunct to traditional time-drawdown techniques of aquifer test analysis of aquifers with anisotropy on the horizontal plane.

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